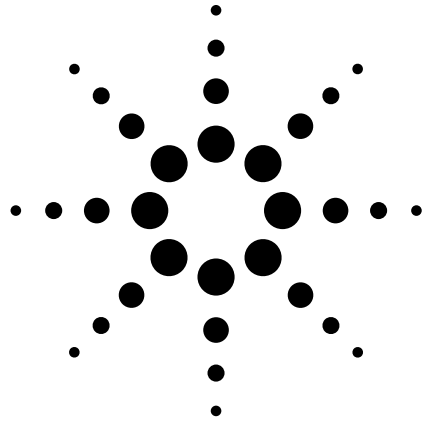# Time Domain Reflectometry Theory

Application Note 1304-2

For Use with
Agilent 86100 Infiniium DCA

Agilent Technologies

Introduction

The most general approach to evaluating the time domain response of any electromagnetic system is to solve Maxwell's equations in the time domain. Such a procedure would take into account all the effects of the system geometry and electrical properties, including transmission line effects. However, this would be rather involved for even a simple connector and even more complicated for a structure such as a multilayer high-speed backplane. For this reason, various test and measurement methods have been used to assist the electrical engineer in analyzing signal integrity.

The most common method for evaluating a transmission line and its load has traditionally involved applying a sine wave to a system and measuring waves resulting from discontinuities on the line. From these measurements, the standing wave ratio (σ) is calculated and used as a figure of merit for the transmission system. When the system includes several discontinuities, however, the standing wave ratio (SWR) measurement fails to isolate them. In addition, when the broadband quality of a transmission system is to be determined, SWR measurements must be made at many frequencies. This method soon becomes very time consuming and tedious.

Another common instrument for evaluating a transmission line is the network analyzer. In this case, a signal generator produces a sinusoid whose frequency is swept to stimulate the device under test (DUT). The network analyzer measures the reflected and transmitted signals from the DUT. The reflected waveform can be displayed in various formats, including SWR and reflection coefficient. An equivalent TDR format can be displayed only if the network analyzer is equipped with the proper software to perform an Inverse Fast Fourier Transform (IFFT). This method works well if the user is comfortable working with s-parameters in the frequency domain. However, if the user is not familiar with these microwave-oriented tools, the learning curve is quite steep. Furthermore, most digital designers prefer working in the time domain with logic analyzers and high-speed oscilloscopes.

When compared to other measurement techniques, time domain reflectometry provides a more intuitive and direct look at the DUT's characteristics. Using a step generator and an oscilloscope, a fast edge is launched into the transmission line under investigation. The incident and reflected voltage waves are monitored by the oscilloscope at a particular point on the line.

This echo technique (see Figure 1) reveals at a glance the characteristic impedance of the line, and it shows both the position and the nature (resistive, inductive, or capacitive) of each discontinuity along the line. TDR also demonstrates whether losses in a transmission system are series losses or shunt losses. All of this information is immediately available from the oscilloscope's display. TDR also gives more meaningful information concerning the broadband response of a transmission system than any other measuring technique.

Since the basic principles of time domain reflectometry are easily grasped, even those with limited experience in high-frequency measurements can quickly master this technique. This application note attempts a concise presentation of the fundamentals of TDR and then relates these fundamentals to the parameters that can be measured in actual test situations. Before discussing these principles further we will briefly review transmission line theory.
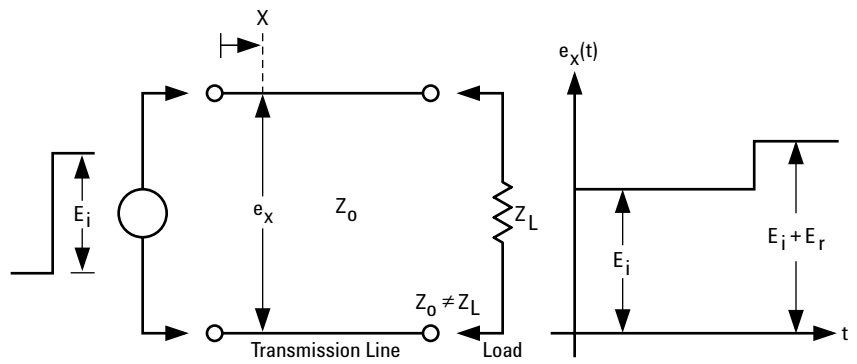

Figure 1. Voltage vs time at a particular point on a mismatched transmission line driven with a step of height E_i

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Propagation on a Transmission Line

The classical transmission line is assumed to consist of a continuous structure of R's, L's and C's, as shown in Figure 2. By studying this equivalent circuit, several characteristics of the transmission line can be determined.

If the line is infinitely long and R, L, G, and C are defined per unit length, then

$$Z_{in} = Z_o \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

where Z_o is the characteristic impedance of the line. A voltage introduced at the generator will require a finite time to travel down the line to a point x. The phase of the voltage moving down the line will lag behind the voltage introduced at the generator by an amount β per unit length. Furthermore, the voltage will be attenuated by an amount α per unit length by the series resistance and shunt conductance of the line. The phase shift and attenuation are defined by the propagation constant γ, where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

and α = attenuation in nepers per unit length
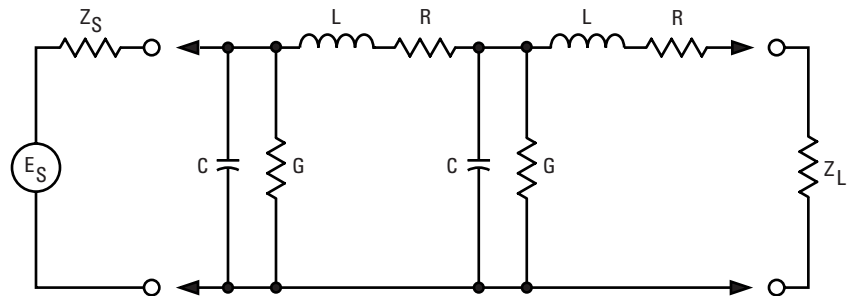β = phase shift in radians per unit length

Figure 2. The classical model for a transmission line.

The velocity at which the voltage travels down the line can be defined in terms of β:

$$\text{Where } v_\rho = \frac{\omega}{\beta} \text{ Unit Length per Second}$$

The velocity of propagation approaches the speed of light, v_c, for transmission lines with air dielectric. For the general case, where e_r is the dielectric constant:

$$v_\rho = \frac{v_c}{\sqrt{e_r}}$$

The propagation constant γ can be used to define the voltage and the current at any distance x down an infinitely long line by the relations

$$E_x = E_{in}e^{-\gamma x} \text{ and } I_x = I_{in}e^{-\gamma x}$$

Since the voltage and the current are related at any point by the characteristic impedance of the line

$$Z_o = \frac{E_{in}e^{-\gamma x}}{I_{in}e^{-\gamma x}} = \frac{E_{in}}{I_{in}} = Z_{in}$$

where E_{in} = incident voltage
I_{in} = incident current

When the transmission line is finite in length and is terminated in a load whose impedance matches the characteristic impedance of the line, the voltage and current relationships are satisfied by the preceding equations.

If the load is different from Z_o, these equations are not satisfied unless a second wave is considered to originate at the load and to propagate back up the line toward the source. This reflected wave is energy that is not delivered to the load. Therefore, the quality of the transmission system is indicated by the ratio of this reflected wave to the incident wave originating at the source. This ratio is called the voltage reflection coefficient, ρ, and is related to the transmission line impedance by the equation:

$$\rho = \frac{E_r}{E_i} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

The magnitude of the steady-state sinusoidal voltage along a line terminated in a load other than Z_o varies periodically as a function of distance between a maximum and minimum value. This variation, called a standing wave, is caused by the phase relationship between incident and reflected waves. The ratio of the maximum and minimum values of this voltage is called the voltage standing wave ratio, σ, and is related to the reflection coefficient by the equation

$$\sigma = \frac{1 + |\rho|}{1 - |\rho|}$$

As has been said, either of the above coefficients can be measured with presently available test equipment. But the value of the SWR measurement is limited. Again, if a system consists of a connector, a short transmission line and a load, the measured standing wave ratio indicates only the overall quality of the system. It does not tell which of the system components is causing the reflection. It does not tell if the reflection from one component is of such a phase as to cancel the reflection from another. The engineer must make detailed measurements at many frequencies before he can know what must be done to improve the broadband transmission quality of the system.

TDR Step Reflection Testing

A time domain reflectometer setup is shown in Figure 3.

The step generator produces a positive-going incident wave that is applied to the transmission system under test. The step travels down the transmission line at the velocity of propagation of the line. If the load impedance is equal to the characteristic impedance of the line, no wave is reflected and all that will be seen on the oscilloscope is the incident voltage step recorded as the wave passes the point on the line monitored by the oscilloscope. Refer to Figure 4.

If a mismatch exists at the load, part of the incident wave is reflected. The reflected voltage wave will appear on the oscilloscope display algebraically added to the incident wave. Refer to Figure 5.
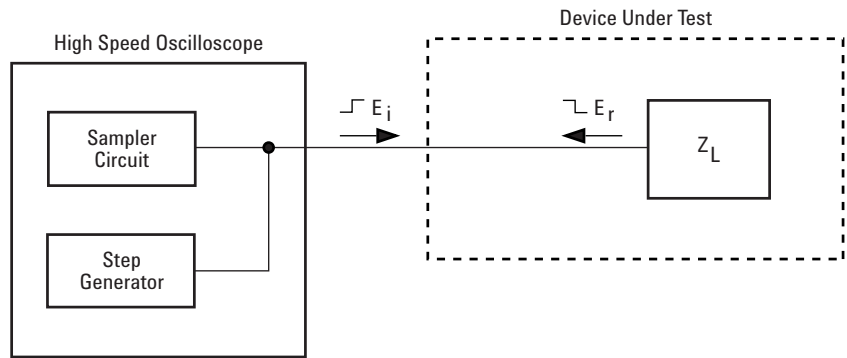
Figure 3. Functional block diagram for a time domain reflectometer
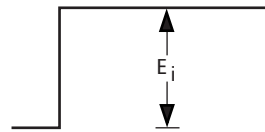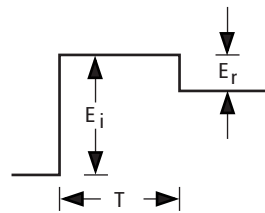
Figure 4. Oscilloscope display when $E_r = 0$

Figure 5. Oscilloscope display when $E_r \neq 0$

Locating Mismatches

The reflected wave is readily identified since it is separated in time from the incident wave. This time is also valuable in determining the length of the transmission system from the monitoring point to the mismatch. Letting D denote this length:

$$D = v_\rho \bullet \frac{T}{2} = \frac{v_\rho T}{2}$$

where v_ρ = velocity of propagation

T = transit time from monitoring point to the mismatch and back again, as measured on the oscilloscope (Figure 5).
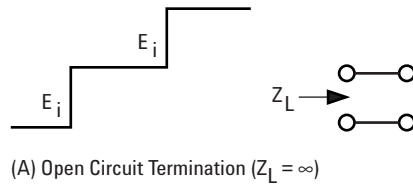
The velocity of propagation can be determined from an experiment on a known length of the same type of cable (e.g., the time required for the incident wave to travel down and the reflected wave to travel back from an open circuit termination at the end of a 120 cm piece of RG-9A/U is 11.4 ns giving v_ρ = 2.1 x 10 cm/sec. Knowing v_ρ and reading T from the oscilloscope determines D. The mismatch is then located down the line. Most TDR's calculate this distance automatically for the user.

Analyzing Reflections

The shape of the reflected wave is also valuable since it reveals both the nature and magnitude of the mismatch. Figure 6 shows four typical oscilloscope displays and the load impedance responsible for each. Figures 7a and 7b show actual screen captures from the 86100A. These displays are easily interpreted by recalling:
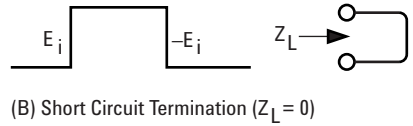
$$\rho = \frac{E_r}{E_i} = \frac{Z_L - Z_O}{Z_L + Z_O}$$

Knowledge of E_i and E_r, as measured on the oscilloscope, allows Z_L to be determined in terms of Z_O, or vice versa. In Figure 6, for example, we may verify that the reflections are actually from the terminations specified.
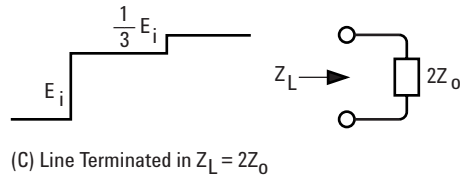
(A) $E_r = E_i$

Therefore $\dfrac{Z_L - Z_0}{Z_L + Z_0} = +1$
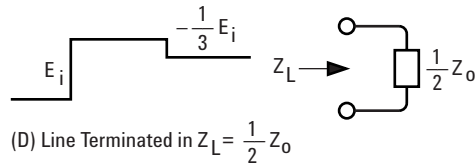
Which is true as $Z_L \longrightarrow \infty$

$\therefore Z =$ Open Circuit

(A) Open Circuit Termination ($Z_L = \infty$)

(B) $E_r = E_i$

Therefore $\dfrac{Z_L - Z_0}{Z_L + Z_0} = -1$

Which is only true for finite Z

When $Z_L = 0$

$\therefore Z =$ Short Circuit

(B) Short Circuit Termination ($Z_L = 0$)

(C) $E_r = +\dfrac{1}{3} E_i$

Therefore $\dfrac{Z_L - Z_0}{Z_L + Z_0} = +\dfrac{1}{3}$

and $Z_L = 2Z_0$

(C) Line Terminated in $Z_L = 2Z_0$

(D) $E_r = -\dfrac{1}{3} E_i$

Therefore $\dfrac{Z_L - Z_0}{Z_L + Z_0} = -\dfrac{1}{3}$

and $Z_L = \dfrac{1}{2} Z_0$

(D) Line Terminated in $Z_L = \dfrac{1}{2} Z_0$

Figure 6. TDR displays for typical loads.

Assuming Z_O is real (approximately true for high quality commercial cable), it is seen that resistive mismatches reflect a voltage of the same shape as the driving voltage, with the magnitude and polarity of E_r determined by the relative values of Z_O and R_L.

Also of interest are the reflections produced by complex load impedances. Four basic examples of these reflections are shown in Figure 8.

These waveforms could be verified by writing the expression for $\rho\,(s)$ in terms of the specific Z_L for each example:

$$\left(\text{i.e., } Z_L = R + sL\, , \ \frac{R}{1 + RC_S}\, , \text{ etc. } \right),$$

multiplying $\rho\,(s)$ by $\dfrac{E_i}{s}$ the transform of a step function of E_i,

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and then transforming this product back into the time domain to find an expression for $e_r(t)$. This procedure is useful, but a simpler analysis is possible without resorting to Laplace transforms. The more direct analysis involves evaluating the reflected voltage at $t = 0$ and at $t = \infty$ and assuming any transition between these two values to be exponential. (For simplicity, time is chosen to be zero when the reflected wave arrives back at the monitoring point.) In the case of the series R-L combination, for example, at $t = 0$ the reflected voltage is $+E_i$. This is because the inductor will not accept a sudden change in current; it initially looks like an infinite impedance, and $\rho = +1$ at $t = 0$. Then current in L builds up exponentially and its impedance drops toward zero. At $t = \infty$, therefore $e_r(t)$ is determined only by the value of R.

$$(\rho = \frac{R - Z_O}{R + Z_O} \text{ When } \tau = \infty)$$

The exponential transition of $e_r(t)$ has a time constant determined by the effective resistance seen by the inductor. Since the output impedance of the transmission line is Z_O, the inductor sees Z_O in series with R, and
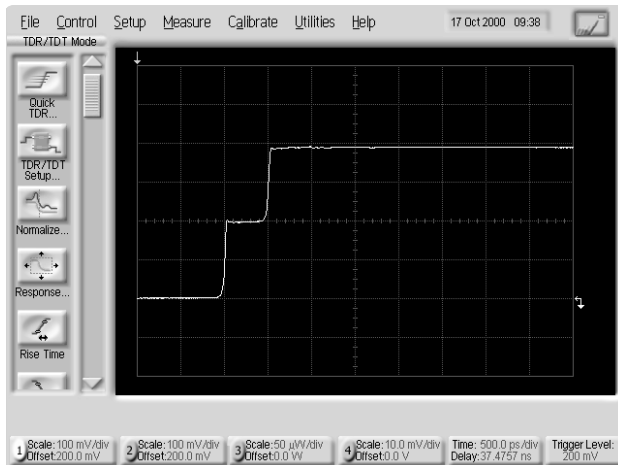
$$\gamma = \frac{L}{R + Z_O}$$

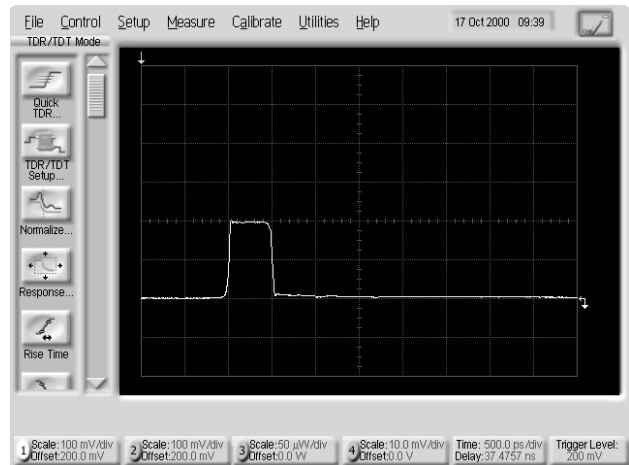**Figure 7a. Screen capture of open circuit termination from the 86100**

Figure 7b. Screen capture of short circuit termination from the 86100

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A Series R–L

$E_i\left[\left(1+\dfrac{R-Z_0}{R+Z_0}\right)+\left(1-\dfrac{R-Z_0}{R+Z_0}\right)e^{-t/\tau}\right]$

E_i

E_i

Where $\tau = \dfrac{L}{R+Z_0}$

$\left(1+\dfrac{R-Z_0}{R+Z_0}\right)E_i$

$Z_L \longrightarrow$ R   L

B Shunt R–C

E_i

$-E_i$

$E_i\left[\left(1+\dfrac{R-Z_0}{R+Z_0}\right)\left(1-e^{-t/\tau}\right)\right]$

$\left(1+\dfrac{R-Z_0}{R+Z_0}\right)E_i$

Where $\tau = \dfrac{Z_0 R}{Z_0+R}\,C$

$Z_L \longrightarrow$ R   C

C Shunt R–L

$\left(\dfrac{R-Z_0}{R+Z_0}\right)E_i$

$E_i\left[\left(1+\dfrac{R-Z_0}{R+Z_0}\right)e^{-t/\tau}\right]$

E_i

Where $\tau = \dfrac{R+Z_0}{RZ_0}\,L$

$Z_L \longrightarrow$ L   R

D Series R–C

$\left(\dfrac{R-Z_0}{R+Z_0}\right)E_i$

$E_i\left[\left(2-\left(1\dfrac{R-Z_0}{R+Z_0}\right)e^{-t/\tau}\right)\right]$

$2E_i$

E_i

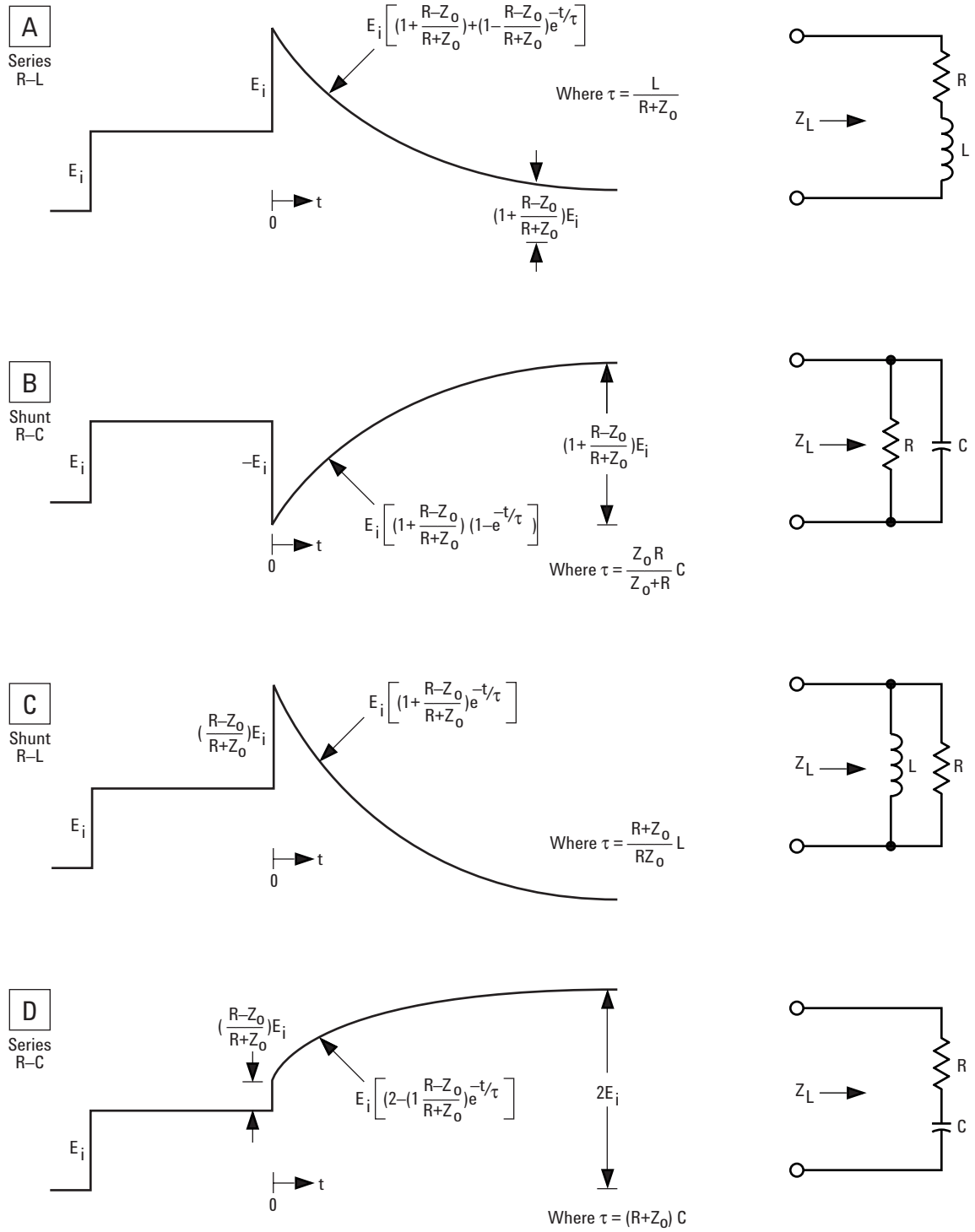Where $\tau = (R+Z_0)\,C$

$Z_L \longrightarrow$ R   C

Figure 8. Oscilloscope displays for complex Z_L.

A similar analysis is possible for the case of the parallel R-C termination. At time zero, the load appears as a short circuit since the capacitor will not accept a sudden change in voltage. Therefore, $\rho = -1$ when $t = 0$. After some time, however, voltage builds up on C and its impedance rises. At $t = \infty$, the capacitor is effectively an open circuit:

$$Z_L = R \quad \text{and} = \frac{R - Z_O}{R + Z_O}$$

The resistance seen by the capacitor is Z_O in parallel with R, and therefore the time constant of the exponential transition of $e_r(t)$ is:

$$\frac{Z_O \, R}{Z_O + R} \, C$$

The two remaining cases can be treated in exactly the same way. The results of this analysis are summarized in Figure 8.

Discontinuities on the Line So far, mention has been made only about the effect of a mismatched load at the end of a transmission line. Often, however, one is not only concerned with what is happening at the load, but also at intermediate positions along the line. Consider the transmission system in Figure 9.

The junction of the two lines (both of characteristic impedance Z_O) employs a connector of some sort. Let us assume that the connector adds a small inductor in series with the line. Analyzing this discontinuity on the line is not much different from analyzing a mismatched termination. In effect, one treats everything to the right of M in the figure as an equivalent impedance in series with the small inductor and then calls this series combination the effective load impedance for the system at the point M. Since the input impedance to the right of M is Z_O, an equivalent representation is shown in Figure 10. The pattern on the oscilloscope is merely a special case of Figure 8A and is shown on Figure 11.
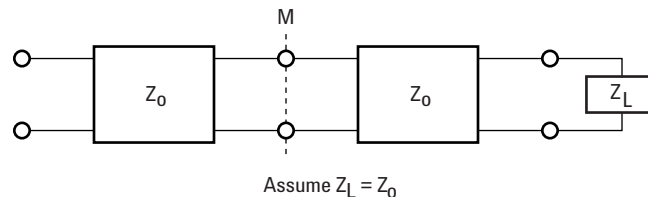

Assume $Z_L = Z_O$

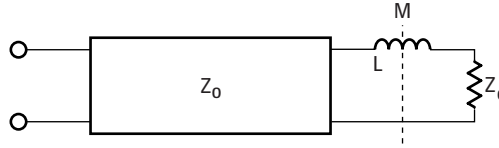**Figure 9. Intermediate positions along a transmission line**
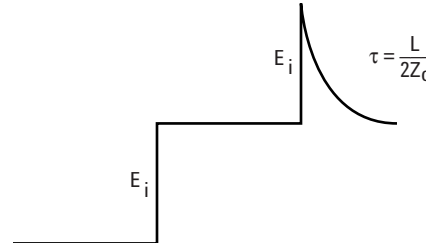
Figure 10. Equivalent representation

Figure 11. Special case of series R-L circuit

Evaluating Cable Loss

Time domain reflectometry is also useful for comparing losses in transmission lines. Cables where series losses predominate reflect a voltage wave with an exponentially rising characteristic, while those in which shunt losses predominate reflect a voltage wave with an exponentially-decaying characteristic. This can be understood by looking at the input impedance of the lossy line.

Assuming that the lossy line is infinitely long, the input impedance is given by:

$$Z_{in} = Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Treating first the case where series losses predominate, G is so small compared to ωC that it can be neglected:

$$Z_{in} = \sqrt{\frac{R + j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \left(1 + \frac{R}{j\omega L} \right)^{1/2}$$

Recalling the approximation $(1 + x)^a \approx (I + ax)$ for $x<1$, Z_{in} can be approximated by:

$$Z_{in} \approx \sqrt{\frac{L}{C}} \left(1 + \frac{R}{j2\omega L} \right) \text{ When } R < \omega L$$

Since the leading edge of the incident step is made up almost entirely of high frequency components, R is certainly less than ωL for $t = 0^+$. Therefore the above approximation for the lossy line, which looks like a simple series R-C network, is valid for a short time after $t = 0$. It turns out that this model is all that is necessary to determine the transmission line's loss.

In terms of an equivalent circuit valid at $t = 0^+$, the transmission line with series losses is shown in Figure 12.
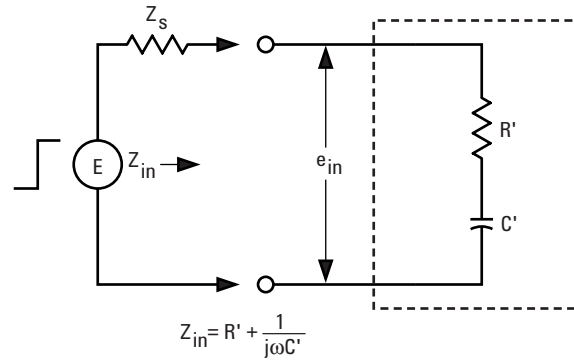

$$Z_{in} = R' + \frac{1}{j\omega C'}$$

Figure 12. A simple model valid at $t = 0^+$ for a line with series losses

The series resistance of the lossy line (R) is a function of the skin depth of the conductor and therefore is not constant with frequency. As a result, it is difficult to relate the initial slope with an actual value of R. However, the magnitude of the slope is useful in comparing conductors of different loss.

A similar analysis is possible for a conductor where shunt losses predominate. Here the input admittance of the lossy cable is given by:

$$Y_{in} = \frac{1}{Z_{in}} = \sqrt{\frac{G + j\omega C}{R + j\omega L}} = \sqrt{\frac{G + j\omega C}{j\omega L}}$$

Since R is assumed small, re-writing this expression for Y_{in}:

$$Y_{in} = \sqrt{\frac{C}{L} \left(1 + \frac{G}{j\omega C} \right)^{1/2}}$$

Again approximating the polynominal under the square root sign:

$$Y_{in} \approx \sqrt{\frac{C}{L} \left(1 + \frac{G}{j2\omega C} \right)} \text{ When } G < \omega C$$

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Going to an equivalent circuit (Figure 13) valid at t = 0$^+$,
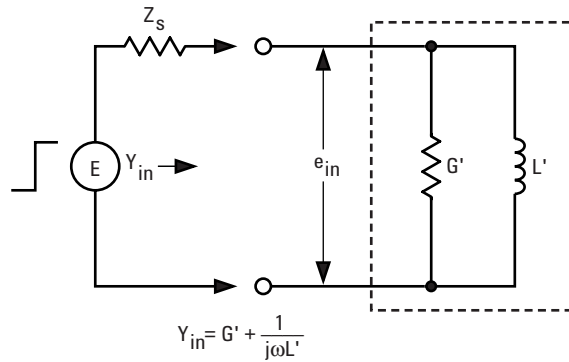

$$Y_{in} = G' + \frac{1}{j\omega L'}$$

Figure 13. A simple model valid at t = 0$^+$ for a line with shunt losses

A qualitative interpretation of why $e_{in}(t)$ behaves as it does is quite simple in both these cases. For series losses, the line looks more and more like an open circuit as time goes on because the voltage wave traveling down the line accumulates more and more series resistance to force current through. In the case of shunt losses, the input eventually looks like a short circuit because the current traveling down the line sees more and more accumulated shunt conductance to develop voltage across.

Multiple Discontinuities One of the advantages of TDR is its ability to handle cases involving more than one discontinuity. An example of this is Figure 14.
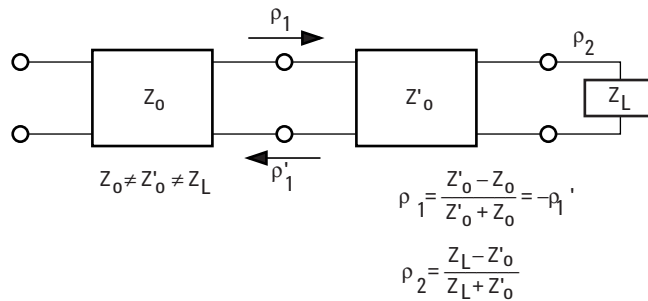

$$Z_0 \neq Z'_0 \neq Z_L$$

$$\rho_1 = \frac{Z'_0 - Z_0}{Z'_0 + Z_0} = -\rho_1'$$

$$\rho_2 = \frac{Z_L - Z'_0}{Z_L + Z'_0}$$

Figure 14. Cables with multiple discontinuities

The oscilloscope's display for this situation would be similar to the diagram in Figure 15 (drawn for the case where $Z_L < Z_0 < Z'_0$:
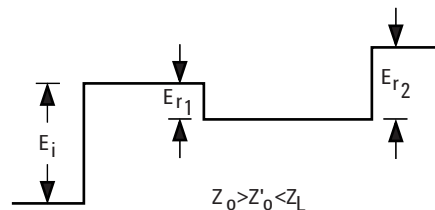

$$Z_0 > Z'_0 < Z_L$$

Figure 15. Accuracy decreases as you look further down a line with multiple discontinuities

It is seen that the two mismatches produce reflections that can be analyzed separately. The mismatch at the junction of the two transmission lines generates a reflected wave, E_r , where

$$E_r = \rho_1 E_i = (\frac{Z'_o - Z_o}{Z'_o + Z_o}) E_i$$

Similarly, the mismatch at the load also creates a reflection due to its reflection coefficient

$$\rho_2 = \frac{Z_L - Z'_o}{Z_L + Z'_o}$$

Two things must be considered before the apparent reflection from Z_L, as shown on the oscilloscope, is used to determine ρ_2. First, the voltage step incident on Z_L is $(1 + \rho_1) E_i$, not merely E_i. Second, the reflection from the load is

$$[\rho_2 (1 + \rho_1) E_i] = E_{r_L}$$

but this is not equal to E_{r_2} since a re-reflection occurs at the mismatched junction of the two transmission lines. The wave that returns to the monitoring point is

$$E_{r_2} = (1 + \rho_1') E_{r_L} = (1 + \rho_1') [\rho_2 (1 + \rho_1) E_i]$$

Since $\rho_1' = -\rho_1$, E_{r_2} may be re-written as:

$$E_{r_2} \quad E_{r_2} = [\rho_2 (1 - \rho_1^2)] E_i$$

The part of E_{r_L} reflected from the junction of

$$E_{r_L} \quad Z'_o \text{ and } Z_o \text{ (i.e., } \rho_1' E_{r_L})$$

is again reflected off the load and heads back to the monitoring point only to be partially reflected at the junction of Z_o' and Z_o. This continues indefinitely, but after some time the magnitude of the reflections approaches zero.

In conclusion, this application note has described the fundamental theory behind time domain reflectometry. Also covered were some more practical aspects of TDR, such as reflection analysis and oscilloscope displays of basic loads. This content should provide a strong foundation for the TDR neophyte, as well as a good brush-up tutorial for the more experienced TDR user.

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