EELE 477
Digital Signal Processing

5b

Implementing FIR Systems

FIR Computation

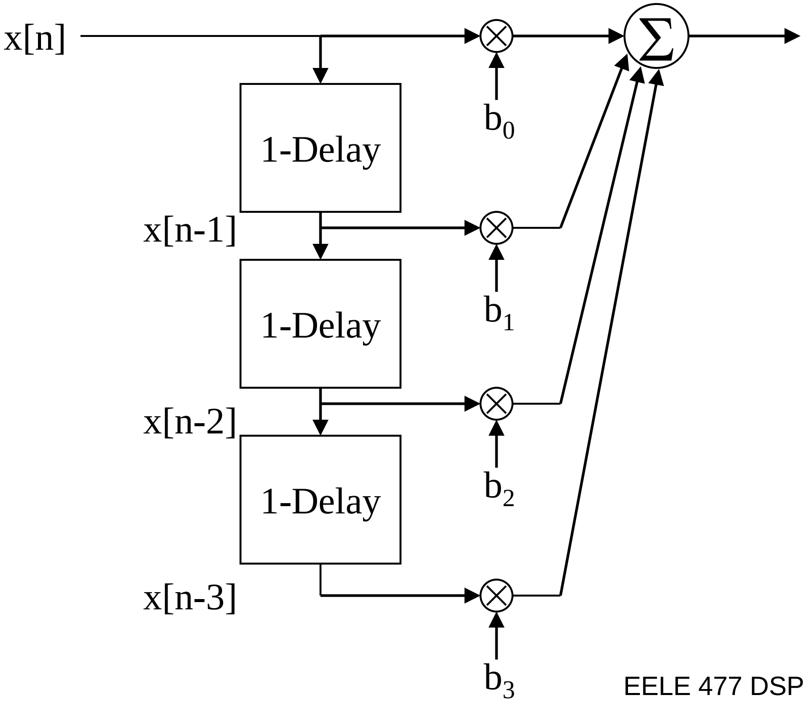
- The general definition of FIR:

$$y[n] = \sum_{k=0}^{M} b_k \cdot x[n-k]$$

- Requires
  - – Delayed values of input $x[n]$
  - – Multiply coefficients b_k
  - – Sum up partial products

FIR Signal Flow Diagram

- Use *unit delay* elements, multipliers, and sum:

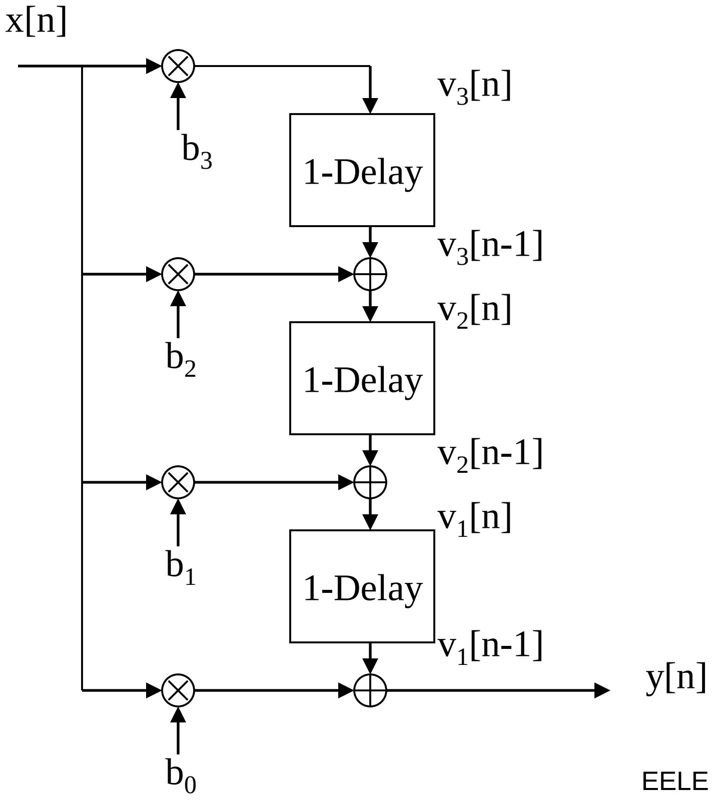


$$y[n] = \sum_{k=0}^{M} b_k \cdot x[n-k]$$

$$= \begin{cases} b_0 \cdot x[n] \\ + b_1 \cdot x[n-1] \\ + b_2 \cdot x[n-2] \\ + b_3 \cdot x[n-3] \end{cases}$$

Another Flow Diagram

- Analyze to show the same result:



$$
\begin{aligned}
y[n] &= b_0 \cdot x[n] + v_1[n-1] \\
&= b_0 \cdot x[n] + b_1 \cdot x[n-1] + v_2[n-2] \\
&= b_0 \cdot x[n] + b_1 \cdot x[n-1] + b_2 \cdot x[n-2] + v_3[n-3] \\
&= b_0 \cdot x[n] + b_1 \cdot x[n-1] + b_2 \cdot x[n-2] + b_3 \cdot x[n-3]
\end{aligned}
$$

Time Invariant

- A time invariant system: delaying the input simply delays the output.

- If *f*() is LTI, then:

$$\text{if } y[n] = f\left(x[n]\right), \rightarrow y[n-n_0] = f\left(x[n-n_0]\right)$$

Linearity

- A linear system: scaling and summing various inputs simply scales and sums the corresponding outputs.

- Linearity implies *superposition*:

$$\text{if } y_1[n] = f\left(x_1[n]\right) \text{ and } y_2[n] = f\left(x_2[n]\right), \text{ then}$$
$$f\left(\alpha \cdot x_1[n] + \beta \cdot x_2[n]\right) = \alpha \cdot y_1[n] + \beta \cdot y_2[n]$$

LTI: Linear Time Invariant

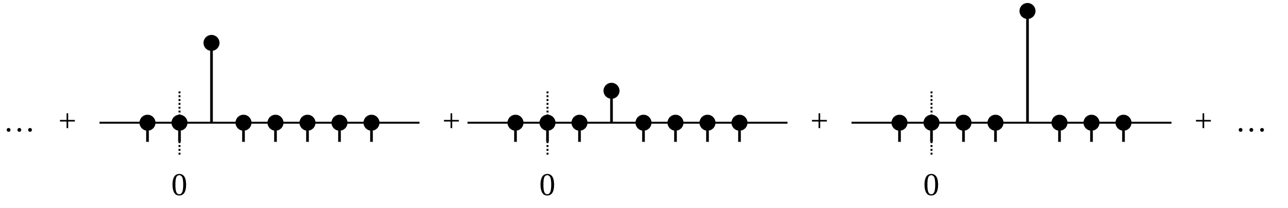
- LTI systems are an important class of systems
- Not *all* useful systems are LTI
- If we know a system is LTI, then we know that it can be fully described by its *unit sample response*, since we can represent the output as a delayed and scaled sum (time shift and superposition).

Convolution and δ[n]

- Recall that:

$$x[n] = \sum_{\text{all } l} x[l] \cdot \delta[n-l]$$

Expresses x[n] as a sum of shifted and scaled impulses:



Convolution (cont.)

- So, applying the shifted and scaled impulses to an LTI system means the output is a set of shifted and scaled impulse responses!

$$y[n] = \sum_{\text{all } l} x[l] \cdot h[n-l]$$

- All LTI systems can be represented this way.

Convolution (cont.)

- Convolution operation is associative, commutative, and distributive
- Cascaded (sequential) LTI systems imply convolution sequence: